Problems from Materials Science provide a rich source of challenges in research directions traditionally associated with pure mathematics, as: Mathematical Analysis, Differential Equations, Geometry, Calculus of Variations. These problems often have a multiscale character; for example, the atomic structure of a material influences its macroscopic properties. Purely continuum models also raise multiscale issues, involving the formation and evolution of patterns as a consequence of loading or phase transformation.

The purpose of this seminar is to study new results in analysis related to the application of Variational Multiscale Methods to systems in Materials Science. Different approaches to coupling of the two extreme scales: the microscopic and the continuum will be at the center of the attention. Other recent advances, such as: emergence of wrinkling patterns driven by loading or growth, prediction of dislocations and fracture, dimension reduction, microstructures and thin films of nematic liquid crystal, will be considered, with an eye on open problems and possible extensions.

As a starting point, we plan to study the following topics:

(1). The geometric rigidity estimate, which paved the way for rigorous derivation of various plate theories through methods of Calculus of Variations.

(2). The method of convex integration, with application to the Born-Infeld equations.

(3). Analysis of the elastic ground states of a nematic glass in the membrane approximation, inducing three-dimensional shapes from flat sheets of material, at the nanoscale all the way to macroscopic objects, including nondevelopable surfaces.

(4). Consider the deformation of a thin elastic film bonded to a thick compliant substrate. Taking a variational viewpoint which views the buckling of the film as a problem of energy-driven pattern formation, one can identify the scaling law of the minimum energy with respect to the physical parameters of the problem, and we prove that a herringbone pattern achieves the optimal scaling.

(5). In several applications, e.g. the theory of plasticity, energy densities with mixed growth appear. Extensions of the result in paper (1) above and of Korn’s inequality,
show that geometric rigidity holds also in $L^p + L^q$ and in the Lorenz interpolation spaces.


The seminar will be self-contained. Knowledge of Lebesgue measure and integration, Lebesgue spaces $L^p$ and basic knowledge of Sobolev spaces $W^{1,p}$, as well as the standard linear algebra material and the very basic differential geometry in $\mathbb{R}^3$ will be assumed.

This seminar will take place in cooperation with Prof. Anja Schlömerkemper. Participation in this seminar can be the basis for a master thesis with her.

**Organizational meeting: April 23, 2013 at 14:15**

Interested students are welcome to contact Dr. Lewicka by email at: lewicka@pitt.edu